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Intermittency at the edge of a stochastically inhibited pattern–forming instability

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Abstract. A layer of ferrofluid under a static vertical magnetic field is submitted to a random vertical vibration which acts as a multiplicative noise on the Rosensweig instability. At low noise amplitude the Rosensweig instability onset is delayed and our experimental results are in good agreement with the theoretical prediction of Lücke *et al.* [1]. For larger noise, a new regime is found where peaks appear and vanish randomly in time. Statistical properties of the temporal evolution of the fluid surface are presented.

PACS. 47.54.+r Pattern selection; pattern formation – 05.45.-a Nonlinear dynamics and nonlinear dynamical systems – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion

Hydrodynamic instabilities can be perturbed by a multiplicative noise in many experimental situations. On the one hand, the control of the forcing parameter can never be perfect and the forcing then fluctuates around the prescribed value. One may think about the temperature regulation of a Rayleigh-Bénard experiment or about the speed control in a Taylor-Couette flow. On the other hand, the bifurcation can occur forced by a fluctuating field. An example of that situation is the dynamo instability where a magnetic field is generated by the velocity field of an electrically conducting liquid. For all known liquid metals, the kinematic viscosity is very small with respect to the magnetic viscosity, thus the velocity is turbulent at dynamo onset. In that case, even with a perfect forcing of the velocity field, the forcing of the magnetic field fluctuates [2].

In order to understand the possible effects of these fluctuations, many studies have been done where the fluctuations are externally prescribed. Two kinds of variations can be identified. In a previous paper, we studied the effect of a parametric modulation of the control parameter of the Rosensweig instability [3]. We showed that the Rosensweig instability is delayed by this modulation. In this paper, we focus on the case where the modulation of the forcing parameter is stochastic. Lücke *et al.* [1] have studied a scalar equation which drives to a supercritical bifurcation in absence of noise. A multiplicative noise delayed the onset of instability. This result has been checked numerically [4]. With an electronic experiment, the same result has been observed for a Hopf bifurcation [5]. For a bifurcating field of higher dimension, modifications of the onset of the instability have been measured in experiments with liquid crystals [6]. For larger noise amplitude, other effects can arise. It has been shown that noise can induce transition that would not occur in the deterministic limit [7]. For a subcritical instability, noise can create new regimes of bistability [8]: in a spatially extended system, the solution switches between two states which are different from the deterministic states. On-off intermittency has been seen in liquid crystal experiments: near a specific point the distribution of the duration of laminar phases is governed by a power law with exponent $-3/2$ [9] .

In this paper, we study the effect of multiplicative noise acting on a static pattern–forming instability: the Rosensweig instability. It appears when a ferrofluid is submitted to a vertical magnetic field and creates static peaks at the fluid surface. The forcing parameter fluctuates because the fluid is vertically vibrated with a random acceleration. For low noise amplitude, the onset of instability is delayed. This is the stabilization of an instability by noise. The onset displacement is correctly predicted by a study of the Mathieu equation including a noise term which describes the evolution of a surface perturbation. At higher noise amplitude, a new effect arises: the surface switches

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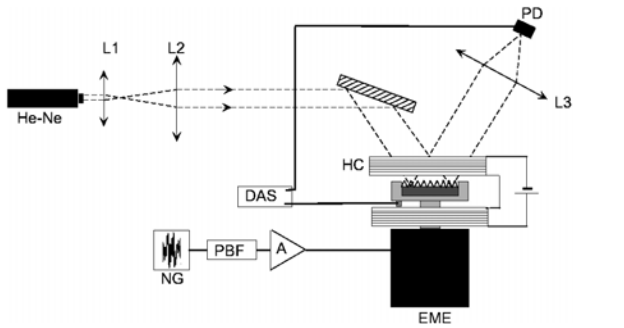

Fig. 1. Sketch of the experimental setup: a circular cell filled with 45 ml of ferrofluid is in the center of a Helmholtz coil (HC) and is vibrated with an electromechanical exciter (EME) driven by a random noise generator (NG). This noise can be filtered (PBF). The peak detection is performed by a laser beam enlarged (with the lenses L1 and L2) and focused by the lens L3 on a photodetector sensor (PD) after reflection on the flat ferrofluid surface. L3 does not focus anymore the beam on the photodetector when Rosensweig peaks appear on the surface.

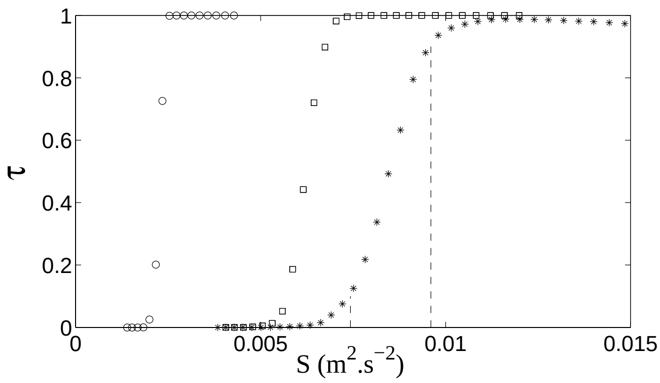

Fig. 2. Evolution of the time τ spent in the flat surface state divided by the total measurement time, as a function of the noise control parameter S. (○) correspond to $B = 159.2$ G where the transition is sharp between 0 and 1. (□) and (∗) correspond to $B = 166$ G and $B = 171$ G respectively. The dashed lines show S_i and S_u (defined in the text) for $B = 171$ G. The spectrum of the noise is in a range [10–100] Hz.

in time between being flat or deformed by the Rosensweig instability.

Our experimental device is drawn in Figure 1. It is almost the one used in [3]. A circular cell, 8.5 cm in inner diameter, containing a ferrofluid layer (APG 521A Ferrotec) 8 mm in depth, is submitted to a magnetic field generated by a Helmhotz coil. We performed the measurement of the ferrofluid density $\rho = 1230$ kg/m^3 and of its surface tension $\gamma = 35 \times 10^{-3}$ N/m known by the measurement of the dispersion relation for the Faraday surface waves. The magnetic properties, given by the manufacturer, are $\chi_i = 1.4$ for the initial magnetic susceptibility and $M_{sat} = 300$ G for the saturation moment. A DC current is supplied to the coil by a stabilized power supply (HP6654A). The cell is put on an electromechanical vibration exciter (BK 4809). This vibrator is driven by a random noise supplied by a digital signal generator (HP8904A). Beforehand, this almost Gaussian white noise is filtered with a pass–band filter (SR650) and amplified (BK 2706 power amplifier). The spectrum of the measured acceleration is therefore almost flat between the cutoff frequencies. The temperature is stabilized around 20 ± 1 °C with a thermal bath (Lauda RC6) and a water circulation. The value of the magnetic field, the rms intensity of the noise and its spectral width are our three control parameters. We measure the cell acceleration with an accelerometer (BK 4393V). The ferrofluid surface state is determined thanks to a He-Ne laser beam. This beam, enlarged over 4 cm^2, is reflected on the flat ferrofluid surface and then is focused on a photodetector sensor (PDA55 ThorLabs). When the ferrofluid surface is deformed, the beam is not focused anymore on the photodetector and the amplitude of the signal almost vanishes.

In the absence of vibration, the surface undergoes an instability for a critical value B_c of the magnetic field. This is the Rosensweig instability that creates an hexagonal pattern of static waves with wavenumber k_c [12]. The magnetic induction threshold is $B_c = 158 \pm 1$ G in agreement with the theoretical value (see below) [10]. When the vibration is added, following [1], we characterize the amplitude of the random acceleration, by the parameter S defined by

$$S = \frac{1}{2\,\pi} \int_0^{\infty} \frac{D(f)}{f_0^2 + f^2}\, df \qquad (1)$$

where f_0 is a viscous cut-off frequency that we will define later and $D(f)$ is the power spectrum of the acceleration. Notice that in absence of applied magnetic field, the flat surface is stable whatever the amplitude of the noisy vibration reached in our experiments.

For a magnetic field slightly above B_c, the instability is inhibited when increasing S. In order to study this transition, we measure τ, the average time spent by the surface being plane divided by the total measurement time. We assume that the surface is flat as soon as the photodetector output signal is greater than $I_f = 0.1\,V$, knowing that when the focusing of the beam is adjusted without vibration the photodetector sensor is almost saturated at 5 V (the precise value taken for I_f does not modify the results presented hereafter). At a precise value S_e, τ changes drastically from 0 to 1 with the control parameter S as shown by the open circle in Figure 2. This means that below S_e the surface is always deformed by the Rosensweig instability and is always plane above S_e as it can be checked by visual inspection. Thus, a stochastic vibration of the vessel inhibits the Rosensweig instability. Even if this instability is known to be subcritical, our measurements are not precise enough to study the corresponding hysteresis cycle *i.e* the same results are obtained if we increase or decrease one of the control parameter (B or S_e). The instability wavenumber k stays roughly constant on the (B, S_e) marginal curve. The growth that one may expect due to the increase of B is too small with respect to the quantization imposed by the lateral boundaries.

This surface behaviour for B close to B_c can be understood with a simple model. For a ferrofluid of viscosity ν, surface tension γ and magnetic susceptibility χ, an eigenmode of wavenumber k of the surface deformation a_k obeys to the equation

$$\ddot{a}_k + d_k \dot{a}_k + (\omega_0^2(B,k) + \zeta(t)k)\, a_k = 0\,, \qquad (2)$$

where the dissipation d_k is $4\,\nu\, k^2$ in the deep layer approximation [11]. The dispersion relation $\omega_0(B,k)$ is given by [12]

$$\omega_0^2(B,k) = g\,k - \frac{\chi^2}{\rho\,\mu_0\,(2+\chi)\,(1+\chi)} B^2\,k^2 + \frac{\gamma}{\rho}\,k^3\,. \qquad (3)$$

The term $\zeta(t)k$ in (2) comes from the effective gravity $g + \zeta(t)$ where the acceleration of the vessel is $\zeta(t)$. It is assumed to be a colored noise characterized by its power spectrum $D(f)$.

In the absence of noise, the surface undergoes an instability at $B = B_c$ and $k = k_c$ for which the pulsation vanishes. This is the Rosensweig instability. For fixed k the behaviour of a_k has been studied by Lücke *et al.* [1]. A simple adaptation of their result shows that the solution $a_k = 0$ is stabilized by noise if

$$S = \frac{1}{2\,\pi}\int_0^\infty \frac{D(f)}{f_0^2 + f^2}\, df \geq -\pi \frac{\omega_0^2(B,k)}{k^2}. \qquad (4)$$

with $f_0 = 2\nu k^2/\pi$. If $B \geq B_c$, many modes k are unstable. We suppose that the surface is plane if all the modes are stabilized, thus if

$$S \geq S_t = -\pi \frac{\omega_0^2(B,k_{min})}{k_{min}^2}\,, \qquad (5)$$

where k_{min} corresponds to the minimum of $\omega_0^2(B,k)/k^2$. In the range where the experiment is done, k_{min} remains close to 600 m^{-1} and this value is used for calculation of f_0. As said before, for B slightly larger than B_c, increasing the noise amplitude stabilizes the flat surface for a particular value of S called S_e, determined with our optical measurement. Figure 3 compares the experimental measurement S_e to the theoretical value S_t given by equation (5) and plotted in full line. The agreement is very good for $B - B_c$ smaller than 4 G. This is true whatever the width of the noise spectrum. Note that the experimental error bars are essentially due to temperature fluctuations. Thus, close to the deterministic onset, the most unstable mode controls the state of the surface and S is the pertinent parameter for describing the noise amplitude.

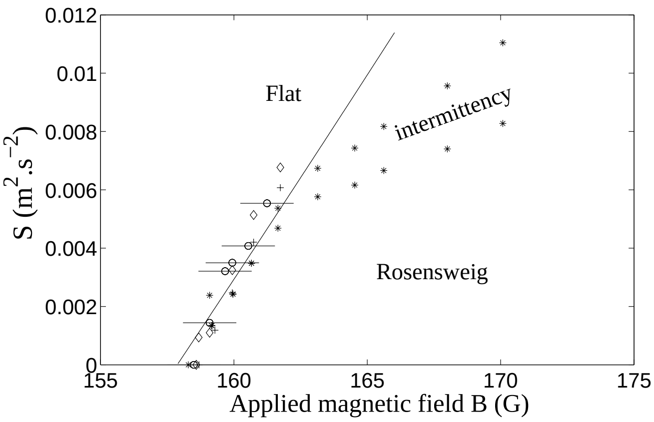

Fig. 3. Stability diagram in the (S–B) plane. The spectral width of the noise is: (∗) 10–100 Hz, (◦) 70–100 Hz, (⋄) 40–300 Hz, (+) 70–300 Hz. The full line is the theoretical value S_t predicted by equation (5).

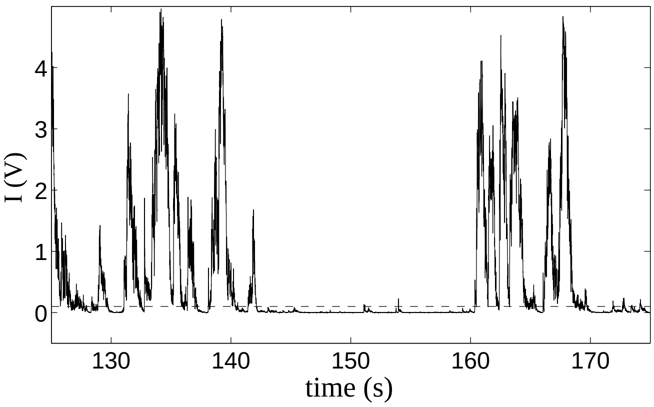

Fig. 4. Temporal trace of the light intensity measured with the photodetector ($B = 163$ G). The dashed line corresponds to $I = 0.1V$ and is chosen as threshold to define the surface state.

An unpredicted dynamical regime appears by increasing the noise level at higher magnetic field. Indeed, for magnetic fields stronger than $B_c + 4$ G, the surface does not undergo a sharp transition from deformed to plane when increasing the noise amplitude. The process is more complex. There is a domain of noise intensity where the Rosensweig peaks appear and disappear randomly in time but they display the same spatial coherence. Figure 4 shows the temporal trace of the light intensity $I(t)$ measured with the photodetector in that case. When $I(t)$ is closed to zero (for instance $I(t) \leq I_f = 0.1$ V, the dashed line on the figure), there are peaks on the surface because the beam is not focused on the sensor, whereas bursts correspond to the flat state of the surface. Characteristic times of $I(t)$ are much larger than the ones of the random acceleration (here the spectrum is almost flat between 10 and 100 Hz). For a given field, the mean duration

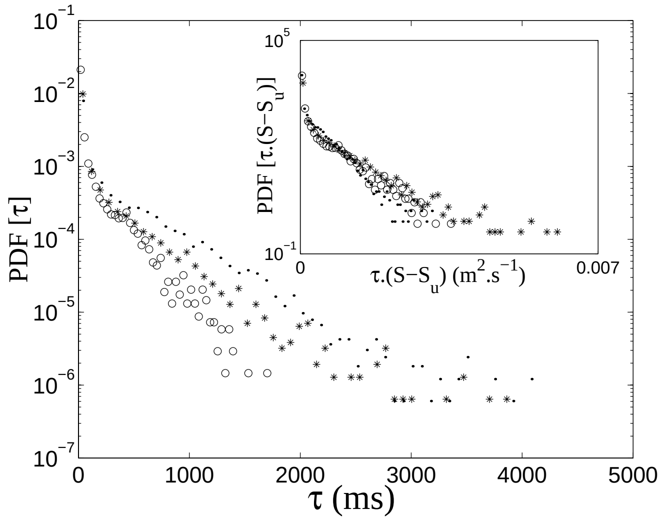

Fig. 5. Probability density function of τ, the time spent by the surface being flat in the intermittent regime ($B = 166$ G) for noise parameters: $S = 5.56 \times 10^{-3}$ (m/s)^2 ($\circ$), $S = 6.61 \times 10^{-3}$ (m/s)^2 ($*$), $S = 6.88 \times 10^{-3}$ (m/s)^2 ($\cdot$). The spectral width of the noise is 10–100 Hz. τ is rescaled by $(S - S_u)$ in the inset. PDF are obtained with 20 000 values of τ.

of the burst increases (respectively decreases) by increasing (resp. decreasing) the noise amplitude and at S_u (resp. S_i) the surface is always flat (resp. the surface is always deformed by the Rosensweig peaks). We call intermittency this regime where the surface switches randomly between two states.

In order to determine experimentally the boundaries $S_u(B)$ and $S_i(B)$, we measure the parameter τ defined above. The evolution of τ with S for different magnetic fields is shown in Figure 2. The open squares and stars illustrate the fact that being zero for small S, τ grows more and more smoothly up to 1 by increasing S. We assume that the Rosensweig regime is reached when $\tau_i \leq 0.1$ and the surface is flat for $\tau_u \geq 0.9$. Other values of τ_u and τ_i do not change qualitatively our results.

We can therefore determine the boundaries between the three different regimes as it is shown in Figure 3 with a noise spectrum in a range [10–100] Hz. Notice that the curve is slightly shifted to the bottom when the smaller frequency of the pass–band filter is increased but this effect is of the same order as the experimental uncertainty.

We plot in Figure 5 the PDF of the time τ that the surface spends being flat, for different values of S. These distributions have clear exponential tails ($\propto \exp(-b\tau)$). Within our range of accuracy, b is proportional to $(S - S_u)^{-1}$, as it can be seen in the inset where are shown the PDF of $(S - S_u)\tau$. The same collapse of the PDF exists with other magnetic fields or frequency ranges of the noise. However for a given value of S, a weak dependence of b on the specific frequency spectrum cannot be excluded here.

We have studied the effect of a multiplicative noise on a pattern forming instability that creates static peaks on the plane surface of a ferrofluid. Slightly above the deterministic onset, noise stabilizes the flat surface. This effect is correctly predicted with a simple model that takes into account only one mode of the surface. For a larger departure from the deterministic onset, large noise amplitude keeps inhibiting the Rosensweig instability but a new effect occurs: instead of a sharp transition from the flat to the spiky interface, there is a regime in which the surface switches randomly between being flat or deformed by the peaks. In that sense we have observed intermittency at the edge of a stochastically inhibited pattern–forming instability.

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